Coordinated Space-time Trajectory Planning and Cyclic Control in Automated Vehicle Zones

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KEYWORDS

Space-time trajectory planning, cyclic control, automated vehicle zone, coordinated control

INTORDUCTION

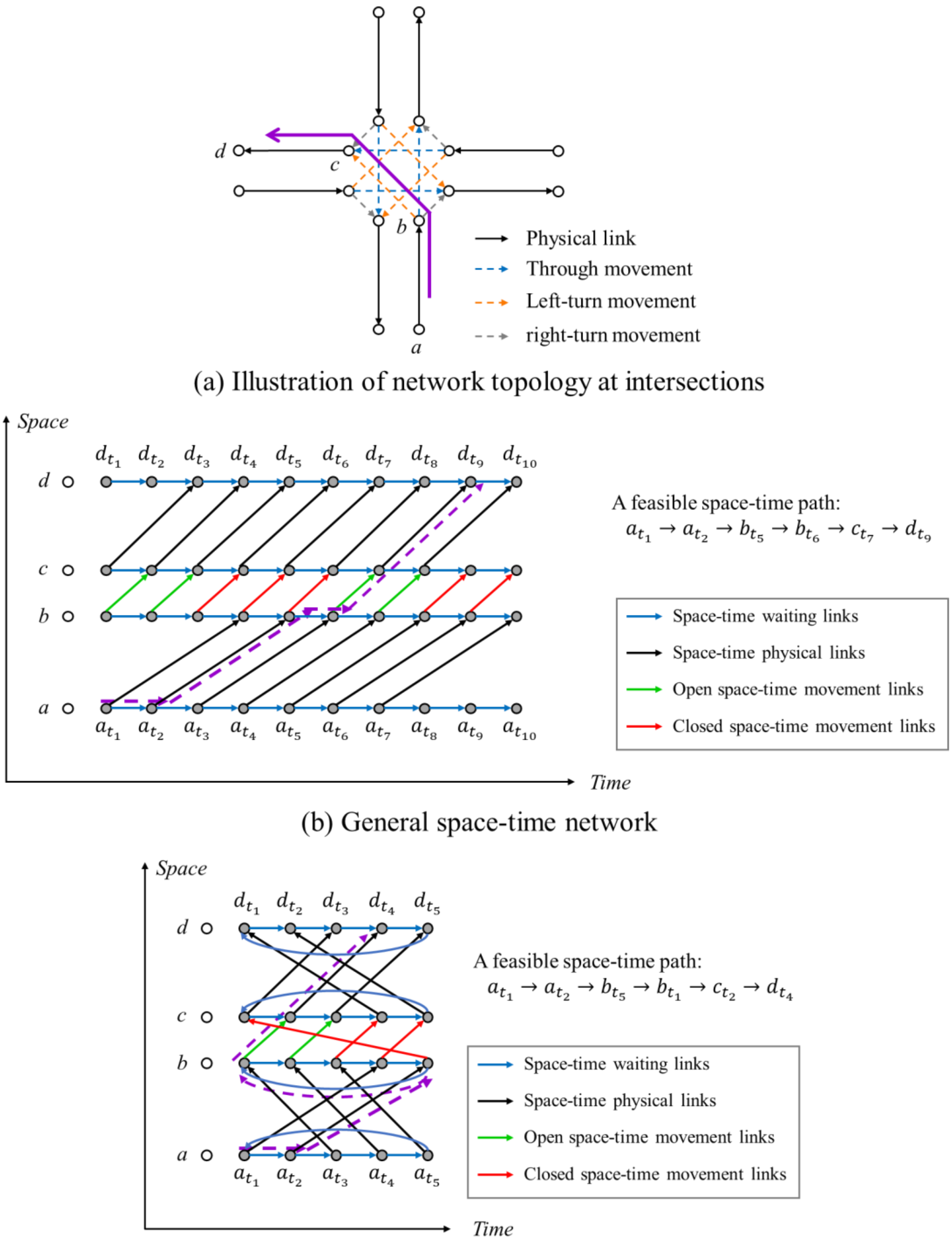
It is envisioned that the combination of Automated Vehicles (AVs) and coordinated network-level control will improve traffic efficiency of the whole system in the future. We find that following topics are mainly paid attention to in related study after careful literature investigation: traffic organization at isolated intersections under pure (connected) AVs or mixed situation (Priemer and Friedrich, 2009; Li et al., 2012; Feng et al., 2015); Vehicle routing considering AVs (Chen at al., 2017; Li et al., 2018); trajectory control of AVs (Lu et al., 2019); joint optimization of signal control and vehicle trajectory at intersections (Guo et al., 2019).

Aiming at developing a coordinated approach between space-time trajectory of AVs and traffic control strategy for fully automated networks, we propose a nested bi-level model to conduct cyclic traffic control and vehicle space-time trajectory planning. Lower-level model is used to plan trajectories under given control strategy, and upper-level is an optimization model of control schemes. Experiments are carried out under different demand levels.

METHODOLOGY

The representation of traffic network adopted in this study is different from traditional one at intersections, which is shown in Figure 1(a). All movements at intersections are represented by links and referred as "movement links" hereafter. Within an intersection, there are through, left-turn, and right-turn movement links. What we usually call a "link" connecting different intersections is termed as a "physical link" here. This representation can easily depict the impact of traffic control strategy.

And then, we build a space-time network to express the relationship of trajectory planning and control strategy. By adding discrete time dimension on spatial network and expanding a spatial node to a series of space-time nodes, we obtain a space-time network. The space-time arcs of networks can be categorized into three types: i) space-time physical links which indicate different entering and exiting time on a same link; ii) space-time movement links which refer to the vehicle movements at intersections at different time step; and iii) space-time waiting links which mean that the traffic flow is hold due to congestion or control strategy. We illustrate these links in Figure 1(b) with a general space-time network first. Taking the path with purple line in Figure 1(a) as an example, there are four spatial nodes along it: *a*, *b*, *c*, and *d*. With discrete time scale $t_1, t_2, \dots, t_{10}$, node a is expanded as $a_{t_1}, a_{t_2}, \dots, a_{10}$, and so are other nodes. It's easy to find a feasible space-time path, such as $a_{t_1} \rightarrow a_{t_2} \rightarrow b_{t_5} \rightarrow b_{t_6} \rightarrow c_{t_7} \rightarrow d_{t_9}$. In this research, we propose a cyclic space-time network. When limiting a control cycle to 5 steps, we obtain the network showed in Figure 1(c). As we conduct cyclic control, there are backward links, which actually point to the nodes of the next cycle. Thus, the space-time path becomes $a_{t_1} \rightarrow a_{t_2} \rightarrow b_{t_5} \rightarrow b_{t_1} \rightarrow c_{t_2} \rightarrow d_{t_4}$.



(a) Illustration of network topology at intersections

(b) General space-time network

(c) Cyclic space-time network

Figure 1. Network topology and cyclic space-time network

In this study, we develop a nested bi-level model to handle the optimization problem. We first build a pre-model on the spatial network to generate an initial traffic control plan. This pre-model can also help to judge if the demands can be handled by suitable control strategy or oversaturation will occur. It is formulated as:

Pre-model (LP-1)

Objective function

$$\min_{x_{ij}^{w}} \sum_{w \in \mathcal{W}} \sum_{(i,j) \in \mathcal{A}} h_{ij} x_{ij}^{w} \tag{1}$$

Constraints

$$\textstyle\sum_{w \in \mathcal{W}} x_{ij}^{w} \leq c_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{2}$$

$$(1+\alpha) \textstyle\sum_{b \in \mathcal{B}_n} y^{n,b} \leq c \left(1 - \frac{W_n}{C_{max}}\right) \qquad \forall n \in \mathcal{N} \tag{3}$$

$$\textstyle\sum_{w \in \mathcal{W}} \frac{x_{ij}^{w}}{l_{ij}} \leq y^{n,b} \qquad \forall n \in \mathcal{N}, b \in \mathcal{B}_n, (i,j) \in \mathcal{A}_m^{n,b} \tag{4}$$

$$\textstyle\sum_{j \in \mathcal{V}} x_{ij}^{w} - \sum_{k \in \mathcal{V}} x_{ki}^{w} = d_i^{w} \qquad \forall i \in \mathcal{V}, w \in \mathcal{W} \tag{5}$$

$$x_{ij}^{w} \geq 0 \qquad \forall (i,j) \in \mathcal{A}, w \in \mathcal{W} \tag{6}$$

For control strategy, we adopt the classical NEMA like strategy for orderly and safe organization of traffic flows, that is, two straight phases and two left-turn phases. We assume that right-turn flows can pass the intersection at any time if the situation allows. To accommodate various demand levels at different intersections, we define a series of candidate cycle lengths in advance, and the initial cycle length will be chosen from this candidate set. However, in pre-model, we just put the maximal cycle length C_{max} in constrain (3) and reselect cycle length according to the pre-model solution.

Taking the initial control strategy as input, lower-level model is formulated as a linear program:

Lower-level model (LP-2)

Objective function

$$\min_{x_{pq,w}^{st}} \sum_{w \in \mathcal{W}^{st}} \sum_{(p,q) \in \mathcal{A}^{st}} h_{pq}^{st} x_{pq,w}^{st} \tag{7}$$

Constraints

$$\textstyle\sum_{w \in \mathcal{W}} x_{pq,w}^{st} \leq c_{pq}^{st} \qquad \forall (p,q) \in \mathcal{A}^{st} \tag{8}$$

$$\sum_{p\in\mathcal{V}^{st}} x_{pq,w}^{st} - \sum_{k\in\mathcal{V}} x_{pq,w}^{st} = d_{p,w}^{st} \qquad \forall p \in \mathcal{V}^{st}, w \in \mathcal{W}^{st} \tag{9}$$

$$x_{pq,w}^{st} = d_{p,w}^{st} \qquad \forall p \in \mathcal{V}^{st}, w \in \mathcal{W}^{st}, (p,q) \in \mathcal{A}_{virtual}^{st} \tag{10}$$

$$x_{pq,w}^{st} \geq 0 \qquad \forall (p,q) \in \mathcal{A}^{st}, w \in \mathcal{W}^{st} \tag{11}$$

To distinguish the notations associated with space-time network from those of the pre-model, all the symbols and variables are marked with st, which is the short hand notation of "space-time". LP-2 can be solved very quickly using some existing solvers, which serves as the foundation of iterative optimization when combining the lower-level and upper-level modes together. To make it clear, we present all the notations used in **LP-1** and **LP-2** in Table 1(a)-(b).

The objective of upper-level model is to search in the reasonable space which consists of the offsets and phase durations at each intersection, and further, to obtain a beneficial control strategy by the feedback from lower-level model. We apply tabu search for upper-level model, which is a heuristic method capable of solving problems with complicated solution space. In pursuit to less complex solution space, we propose an optimization framework (see Table 1(c)) and decide to optimize the offsets and phase durations successively. Both traffic control strategy and vehicle routing solution are acquired after several rounds.

Table 1. Notations and coordinated optimization steps

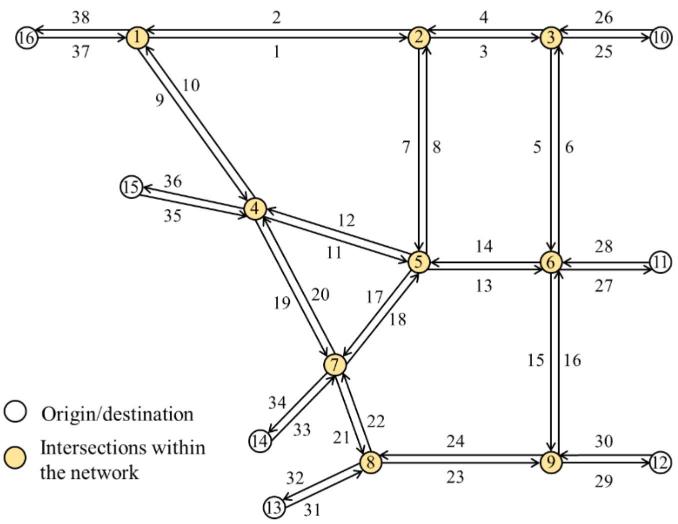
(a) Notations for the pre-model	
Sets	
$\mathcal{V}$	Set of all nodes in the road network
$\mathcal{N}$	Set of all intersections in the network
$\mathcal{A}$	Set of all links in the network, including physical and movement links
$\mathcal{B}_n$	Set of all phases at intersection n
$\mathcal{A}_m^{n,b}$	Set of straight and left-turn movement links inside intersection $n \in \mathcal{N}$ which are open during phase $b \in \mathcal{B}_n$
$\mathcal{W}$	Set of all O-D pairs in network
Decision variables	
x_{ij}^w	Traffic flow on link $(i,j) \in \mathcal{A}$ of O-D pair w, where $i,j \in \mathcal{V}$
$y^{n,b}$	Maximal flow rate at intersection n during phase b
Inputs	
d_w	Travel demand rate of O-D pair w with the origin r_w and associated destination s_w
d_i^w	Virtual demand related to node i, $d_i^w = d_w$ if $i = r_w$; $d_i^w = -d_w$ if $i = s_w$; otherwise $d_i^w = 0$
c	Saturation flow rate with the unit of veh/step/lane
l_{ij}	The number of lanes of link $(i,j) \in \mathcal{A}$
c_{ij}	Saturation flow rate of link (i,j), $c_{ij} = c * l_{ij}$
h_{ij}	Free flow travel time on link $(i,j) \in \mathcal{A}_p$
α	A positive number to guarantee the control robustness
C_{max}	Maximal cycle length of the network
W_n	Total wasted time at intersection n due to phase transition

(b) Notations for the lower-level space-time model	
Sets	
$\mathcal{V}^{st}$	Set of all nodes in the space-time network
$\mathcal{A}^{st}$	Set of all links in the space-time network
$\mathcal{A}_{virtual}^{st}$	Set of all virtual links in the space-time network
$\mathcal{W}^{st}$	Set of all O-D pairs in space-time network
Decision variables	
$x_{pq,w}^{st}$	Traffic flow on link $(p,q) \in \mathcal{A}$ of O-D pair w^{st}, where $p,q \in \mathcal{V}^{st}$
Inputs	
d_w^{st}	Travel demand rate of O-D pair w^{st} with the origin r_w^{st} and associated destination s_w^{st}
$d_{p,w}^{st}$	Virtual demand related to node p, $d_p^w = d_w^{st}$ if $p = r_w^{st}$; $d_{p,w}^{st} = -d_w^{st}$ if $p = s_w^{st}$; otherwise $d_{p,w}^{st} = 0$
c	Saturation flow rate with the unit of veh/step/lane
l_{pq}^{st}	The number of lanes of link $(p,q) \in \mathcal{A}$
c_{pq}^{st}	Saturation flow rate of link (p,q), $c_{pq}^{st} = c * l_{pq}^{st}$
h_{pq}^{st}	Free flow travel time on space-time link (p,q)

(c) Steps of proposed nested bi-level coordinated optimization model	
Step 0.	(Initialization) Solve the pre-model to get an initial traffic control strategy.
Step 1.	(Inner loop) Conduct N_1 times iterative interaction between lower-level and upper-level model. N_1 is a pre-defined parameter.
Step 2.	(Outer loop) Adjust the traffic control strategy according to results given by last solution of lower-level model in the inner loop.
Step 3.	(Terminate condition) If terminate condition is satisfied, terminate the procedure and output results. The condition can be: (1) for two successive outer loops the improvement of the objective function of lower-level model is no more than ε (a predetermined subtle value); (2) maximal number of outer loops N_2 is reached.

RESULTS

We test the proposed method on a small network with nine intersections. The network topology and link lengths, as well as OD demand we used in this network are displayed in Figure 2(a). The control cycle is set as 90 s and each time step 3 s. Candidate cycle length of intersections is $\{30\ s, 45\ s, 90\ s\}$. Besides, both the travel on movement links and waste time due to phase transition are defined as one step. The saturation flow rate for AVs is set to 2400 vehs/hour/lane, i.e., 2 vehs/lane per time step. Traffic demand is set the same for all eight OD pairs, and seven demand levels are tested: $\{120, 240, 360, 480, 600, 720, 840\}\ vehs/h$. The optimization results are given in Figure 2(b).

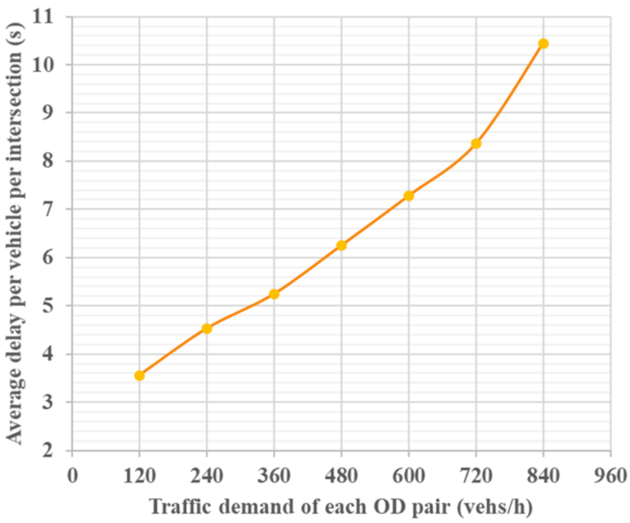


Physical link	Length (m)
1/2	1020
3/4/13/14/17/18	480
5/6/7/8	840
9/10	780
11/12	600
15/16	700
19/20	660
21/22	340
23/24	660
25-38	300

O	D
16	12
13	10
11	16
14	11
10	15
12	13
12	15
14	10

O: Origin
D: destination

(a) Test network and O-D demand



(b) Experiment results

Figure 2. Testing network and results

According to Figure 2(b), the average delay various from 3 to 11 seconds under different demand levels. With small demands on the network, the average delay is near to zero, which is a fine result. When traffic demands become more and more, the average delay time also increases, but still at an acceptable level.

CONCLUSION

In this study, we propose a nested bi-level optimization model to iterative optimize traffic control strategy and vehicle space-time trajectories for fully automated networks. Experiment results show good performance of the proposed method, which makes it possible to conduct network-level coordination of road systems and AVs. However, it is noticed that in our built model, all the variables are treated as continuous ones, which is not realistic in real-life. So, suitable methods to translate the optimization results to real-life discrete vehicle routing schemes needs to be explored.

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